Elementary proof that \( e \) is irrational

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The following variation on the usual proof of the irrationality of \( e \) is perhaps slightly simpler. Suppose that \( e \) is rational, say \( e = a/b \). Then

\[
\frac{b}{a} = \frac{1}{e} = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!}
\]

and multiplication by \((-1)^{a+1}a!\) and transposition of terms gives

\[
(-1)^{a+1} \left\{ b(a - 1)! - \sum_{n=0}^{a} \frac{(-1)^n a!}{n!} \right\}
\]

\[
= \frac{1}{(a + 1)} - \frac{1}{(a + 1)(a + 2)} + \frac{1}{(a + 1)(a + 2)(a + 3)} - \cdots
\]

The right side has a value between 0 and 1 since the alternating series clearly converges to a value between its first term and the sum of its first two terms. But the left side is an integer, so we have a contradiction.